1 Practice Problems (Pigeonhole principle)

1. Recall that a regular icosahedron is a convex polyhedron having 12 vertices and 20 faces; the faces are congruent equilateral triangles. On each face of a regular icosahedron is written a nonnegative integer such that the sum of all 20 integers is 39. Show that there are two faces that share a vertex and have the same integer written on them.

2. Draw the diagonals of a 21-gon. Prove that at least one angle of less than 1 degree is formed.

3. Inside the unit square lie several circles the sum of whose circumferences is equal to 10. Prove there exist infinitely many lines each of which intersects at least four of the circles.

4. The points of the plane are colored by finitely many colors. Prove that one can find a rectangle with vertices of the same color.

5. (hw) Show that any convex polyhedron has two faces with the same number of edges.

6. (hw) If each square of a 3-by-7 chessboard is colored either black or white, then the board must contain a rectangle consisting of at least four squares whose corner squares are either all white or all black.

7. (hw) There are 33 students in the class and sum of their ages 430 years. Is it true that one can find 20 students in the class such that sum of their ages greater 260.

8. (hw) 14 integers are chosen from $\{1, 2, \ldots, 28\}$. Prove that there exist four of the chosen integers which can be split into two groups of two with the same sum.